Question	Scheme	Marks	AOs
1(a)(i)	$50x^2+38x+9\equiv A(5x+2)(1-2x)+B(1-2x)+C(5x+2)^2$ $\Rightarrow B=\ldots$ or $C=\ldots$	M1	1.1b
	$B=1$ and $C=2$	A1	1.1b
(a)(ii)	E.g. $x=0$ $x=0\Rightarrow 9=2A+B+4C$ $\Rightarrow 9=2A+1+8\Rightarrow A=\ldots$	M1	2.1
	$A=0$*	A1*	1.1b
		(4)	
(b)(i)	$\dfrac{1}{(5x+2)^2}=(5x+2)^{-2}=2^{-2}\left(1+\dfrac{5}{2}x\right)^{-2}$ or $(5x+2)^{-2}=2^{-2}+\ldots$	M1	3.1a
	$\left(1+\dfrac{5}{2}x\right)^{-2}=1-2\left(\dfrac{5}{2}x\right)+\dfrac{-2(-2-1)}{2!}\left(\dfrac{5}{2}x\right)^2+\ldots$	M1	1.1b
	$2^{-2}\left(1+\dfrac{5}{2}x\right)^{-2}=\dfrac{1}{4}-\dfrac{5}{4}x+\dfrac{75}{16}x^2+\ldots$	A1	1.1b
	$\dfrac{1}{(1-2x)}=(1-2x)^{-1}=1+2x+\dfrac{-1(-1-1)}{2!}(2x)^2+.$	M1	1.1b
	$\dfrac{1}{(5x+2)^2}+\dfrac{2}{1-2x}=\dfrac{1}{4}-\dfrac{5}{4}x+\dfrac{75}{16}x^2+\ldots+2+4x+8x^2+\ldots$	dM1	2.1
	$=\dfrac{9}{4}+\dfrac{11}{4}x+\dfrac{203}{16}x^2+\ldots$	A1	1.1b
(b)(ii)	$\lvert x\rvert<\dfrac{2}{5}$	B1	2.2a
		(7)	
			(11 marks)

Notes

(a)(i)

M1: **Uses a correct identity** and makes progress using an appropriate strategy (e.g. sub $x=\frac{1}{2}$) to find a value for B or C. May be implied by one correct value (cover up rule).

A1: Both values correct

(a)(ii)

M1: Uses an appropriate method to establish an equation connecting A with B and/or C and uses their values of B and/or C to find a suitable equation in A.

Amongst many different methods are:

Compare terms in x^2 $\Rightarrow 50=-10A+25C$ which would be implied by $50=-10A+25\times\text{"2"}$

Compare constant terms or substitute $x=0\Rightarrow 9=2A+B+4C$ implied by $9=2A+1+4\times 2$

A1*: Fully correct proof with no errors.

Note: The second part is a proof so it is important that a suitable proof/show that is seen.

Candidates who write down 3 equations followed by three answers (with no working) will score M1 A1 M0 A0 (b)(i)

M1: Applies the key steps of writing $\frac{1}{(5x+2)^2}$ as $(5x+2)^{-2}$ and takes out a factor of 2^{-2} to form an expression of the form $(5x+2)^{-2} = 2^{-2}(1+*x)^{-2}$ where * is not 1 or 5

Alternatively uses direct expansion to obtain $2^{-2} + \ldots$

M1: Correct attempt at the binomial expansion of $(1+*x)^{-2}$ up to the term in x^2

Look for $1+(-2)*x+\frac{(-2)(-3)}{2}*x^2$ where * is not 5 or 1.

Condone sign slips and lack of $*^2$ on term 3. ....

Alt Look for correct structure for 2nd and 3rd terms by direct expansion. See below

A1: For a fully correct expansion of $(2+5x)^{-2}$ which may be unsimplified. This may have been combined with their '*B*'

A direct expansion would look like $(2+5x)^{-2} = 2^{-2}+(-2)2^{-3}\times 5x+\frac{(-2)(-3)}{2}2^{-4}\times(5x)^2$

M1: Correct attempt at the binomial expansion of $(1-2x)^{-1}$

Look for $1+(-1)*x+\frac{(-1)(-2)}{2}*x^2$ where * is not 1

dM1: Fully correct strategy that is dependent on the previous **TWO** method marks.
There must be some attempt to use their values of *B* and *C*

A1: Correct expression or correct values for *p*, *q* and *r*.

(b)(ii)

B1: Correct range. Allow also other forms, for example $-\frac{2}{5}<x<\frac{2}{5}$ or $x \in \left(-\frac{2}{5},\frac{2}{5}\right)$

Do not allow multiple answers here. The correct answer must be chosen if two answers are offered

Question	Scheme	Marks	AOs
2(a)	$\sqrt{4-9x}=2(1\pm...)^{\frac{1}{2}}$	B1	1.1b
	$\left(1-\text{"}\frac{9x}{4}\text{"}\right)^{\frac{1}{2}}=...+\frac{\frac{1}{2}\times\left(-\frac{1}{2}\right)\left(\text{"}-\frac{9x}{4}\text{"}\right)^2}{2!}$ or $...+\frac{\frac{1}{2}\times\left(-\frac{1}{2}\right)\times\left(-\frac{3}{2}\right)\left(\text{"}-\frac{9x}{4}\text{"}\right)^3}{3!}$	M1	1.1b
	$1+\frac{1}{2}\times\left(-\frac{9x}{4}\right)+\frac{\frac{1}{2}\times\left(-\frac{1}{2}\right)\left(-\frac{9x}{4}\right)^2}{2!}+\frac{\frac{1}{2}\times\left(-\frac{1}{2}\right)\times\left(-\frac{3}{2}\right)\left(-\frac{9x}{4}\right)^3}{3!}$	A1	1.1b
	$\sqrt{4-9x}=2-\frac{9x}{4}-\frac{81x^2}{64}-\frac{729x^3}{512}$	A1	1.1b
		(4)	
(b)	States that the approximation will be an **<u>overestimate</u>** since all terms (after the first one) in the expansion are negative (since $x>0$)	B1	3.2b
		(1)	
			(5 marks)
Notes:			

(a)

B1: Takes out a factor of 4 and writes $\sqrt{4-9x}=2(1\pm...)^{\frac{1}{2}}$ or $\sqrt{4}(1\pm...)^{\frac{1}{2}}$ or $4^{\frac{1}{2}}(1\pm...)^{\frac{1}{2}}$

M1: For an attempt at the binomial expansion of $(1+ax)^{\frac{1}{2}}$ $a\neq 1$ to form term 3 or term 4 with the correct structure. Look for the correct binomial coefficient multiplied by the corresponding power of x e.g.

$$\frac{\left(\frac{1}{2}\right)\left(\frac{1}{2}-1\right)}{2!}(...x)^2 \quad \textbf{or} \quad \frac{\left(\frac{1}{2}\right)\left(\frac{1}{2}-1\right)\left(\frac{1}{2}-2\right)}{3!}(...x)^3 \quad \text{where} ...\neq 1$$

Condone missing or incorrect brackets around the *x* terms but the binomial coefficients must be correct. Allow 2! and/or 3! or 2 and/or 6. Ignore attempts to find more terms.

Do not allow notation such as $\begin{pmatrix}\frac{1}{2}\\1\end{pmatrix}, \begin{pmatrix}\frac{1}{2}\\2\end{pmatrix}$ unless these are interpreted correctly.

A1: Correct expression for the expansion of $\left(1-\frac{9x}{4}\right)^{\frac{1}{2}}$ e.g.

$$1+\frac{1}{2}\times\left(-\frac{9x}{4}\right)+\frac{\frac{1}{2}\times\left(\frac{1}{2}-1\right)\left(\pm\frac{9x}{4}\right)^2}{2!}+\frac{\frac{1}{2}\times\left(\frac{1}{2}-1\right)\times\left(\frac{1}{2}-2\right)\left(-\frac{9x}{4}\right)^3}{3!}$$

which may be left unsimplified as shown but the bracketing must be correct unless any missing brackets are implied by subsequent work. If the 2 outside this expansion is only partially applied to this expansion then score A0 but if it is applied to all terms this A1 can be implied.

OR at least 2 correct simplified terms **for the final expansion** from, $-\frac{9x}{4}$, $-\frac{81x^2}{64}$, $-\frac{729x^3}{512}$

A1: $\sqrt{4-9x}=2-\frac{9x}{4}-\frac{81x^2}{64}-\frac{729x^3}{512}$ oe and condone e.g. $2+\frac{-9x}{4}-\frac{81x^2}{64}-\frac{729x^3}{512}$

Allow equivalent mixed numbers and/or decimals for the coefficients e.g.:

$\left(\frac{9}{4}, 2\frac{1}{4}, 2.25\right), \left(\frac{81}{64}, 1\frac{17}{64}, 1.265625\right), \left(\frac{729}{512}, 1\frac{217}{512}, 1.423828125\right)$

Ignore any extra terms if found. Allow terms to be "listed" and apply isw once a correct expansion is seen. Allow recovery if applicable e.g. if an "x" is lost then "reappears".

<u>Direct expansion in (a) can be marked in a similar way:</u>

$$\sqrt{4-9x} = (4-9x)^{\frac{1}{2}} = 4^{\frac{1}{2}} + \left(\frac{1}{2}\right)4^{-\frac{1}{2}} \times (-9x)^1 + \left(\frac{1}{2}\right)\left(\frac{1}{2}-1\right)4^{-\frac{3}{2}} \times \frac{(-9x)^2}{2!} + \left(\frac{1}{2}\right)\left(\frac{1}{2}-1\right)\left(\frac{1}{2}-2\right)4^{-\frac{5}{2}} \times \frac{(-9x)^3}{3!}$$

B1: For 2 or $\sqrt{4}$ or $4^{\frac{1}{2}}$ as the constant term in the expansion.

M1: Correct form for term 3 or term 4.

E.g. $\left(\frac{1}{2}\right)\left(-\frac{1}{2}\right) \times \frac{(...x)^2}{2!}$ or $\left(\frac{1}{2}\right)\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right) \times \frac{(...x)^3}{3!}$ where $... \neq 1$

Condone missing brackets around the x terms but the binomial coefficients must be correct. Allow 2! and/or 3! or 2 and/or 6. Ignore attempts to find more terms.

Do not allow notation such as $\begin{pmatrix} \frac{1}{2} \\ 1 \end{pmatrix}, \begin{pmatrix} \frac{1}{2} \\ 2 \end{pmatrix}$ unless these are interpreted correctly.

A1: Correct expansion (unsimplified as above)

OR at least 2 correct simplified terms from, $-\frac{9x}{4}, \quad -\frac{81x^2}{64}, \quad -\frac{729x^3}{512}$

A1: $\sqrt{4-9x} = 2 - \frac{9x}{4} - \frac{81x^2}{64} - \frac{729x^3}{512}$ oe and condone e.g. $2 + \frac{-9x}{4} - \frac{81x^2}{64} - \frac{729x^3}{512}$

Allow equivalent mixed numbers and/or decimals for the coefficients e.g.:

$\left(\frac{9}{4}, 2\frac{1}{4}, 2.25\right), \left(\frac{81}{64}, 1\frac{17}{64}, 1.265625\right), \left(\frac{729}{512}, 1\frac{217}{512}, 1.423828125\right)$

Ignore any extra terms if found. Allow terms to be "listed" and apply isw once a correct expansion is seen. Allow recovery if applicable e.g. if an "x" is lost then "reappears".

(b)

B1: States that the approximation will be an **<u>overestimate</u>** due to the fact that all terms (after the first one) in the expansion are negative or equivalent statements e.g.

- Overestimate because the terms are negative
- Overestimate as the terms are being taken away (from 2)

Condone "overestimate as every term is negative"

If you think a response is worthy of credit but are unsure then use Review.

This mark depends on having obtained an expansion in (a) of the form

$k - px - qx^2 - rx^3 \quad k, p, q, r > 0$ but note that if e.g. one of the algebraic terms is zero or was "lost" or there are extra negative terms this mark is still available.

....

Question	Scheme	Marks	AOs
3(a)	$3^{-2}\left(1+\frac{x}{3}\right)^{-2}=3^{-2}(1+...x+...x^2)$	M1	1.1b
	$(-2)\left(\frac{x}{3}\right)$ or $\frac{(-2)(-3)}{2!}\left(\frac{x}{3}\right)^2$	M1	1.1b
	$\left(1+\frac{x}{3}\right)^{-2}=1+(-2)\left(\frac{x}{3}\right)+\frac{(-2)(-3)}{2!}\left(\frac{x}{3}\right)^2$	A1	1.1b
	$3^{-2}\left(1+\frac{x}{3}\right)^{-2}=\frac{1}{9}-\frac{2x}{27}+\frac{x^2}{27}$	A1	2.1
		(4)	

(a)

M1: Attempts a binomial expansion by taking out a factor of 3^{-2} or $\frac{1}{3^2}$ or $\frac{1}{9}$ and achieves at least the first 3 terms in their expansion. May be seen separately e.g. $\frac{1}{9}$ and $(1+...x+...x^2)$

M1: A correct method to find either the x or the x^2 term unsimplified.

Award for $(-2)(kx)$ or $\frac{(-2)(-2-1)}{2!}(kx)^2$ where $k\neq 1$. Condone invisible brackets.

A1: For a correct unsimplified or simplified expansion of $\left(1+\frac{x}{3}\right)^{-2}$ e.g. $=1+(-2)\left(\frac{x}{3}\right)+\frac{(-2)(-3)}{2!}\left(\frac{x}{3}\right)^2-...$ or $1-\frac{2x}{3}+\frac{x^2}{3}-...$ Do not condone missing brackets unless they are implied by subsequent work.

Condone $\left(-\frac{x}{3}\right)^2$ for $\left(\frac{x}{3}\right)^2$

Also allow this mark for 2 correct simplified terms from $\frac{1}{9}-\frac{2x}{27}+\frac{x^2}{27}$ with both method marks scored.

A1: $\frac{1}{9}-\frac{2x}{27}+\frac{x^2}{27}$ cao Allow terms to be listed. Ignore any extra terms. Isw once a correct **simplified** answer is seen.

<u>**Direct expansion, if seen, should be marked as follows:**</u>

$$\left((3+x)^{-2}=3^{-2}-2\times 3^{-3}\times x+\frac{-2(-2-1)}{2!}\times 3^{-4}\times x^2\right)$$

M1: For $(3+x)^{-2}=3^{-2}+3^{-3}\times\alpha x+3^{-4}\times\beta x^2$

M1: A correct method to find either the x or the x^2 term unsimplified.

Award for $(-2)\times 3^{-3}x$ or $\frac{(-2)(-2-1)}{2!}\times 3^{-4}x^2$. Condone invisible brackets.

A1: For a correct unsimplified or simplified expansion of $(3+x)^{-2}$ e.g. $3^{-2}-2\times 3^{-3}\times x+\frac{-2(-2-1)}{2!}\times 3^{-4}\times x^2$

Also award for at least 2 correct simplified terms from $\frac{1}{9}-\frac{2x}{27}+\frac{x^2}{27}$ with both method marks scored.

A1: $\frac{1}{9}-\frac{2x}{27}+\frac{x^2}{27}$ cao Allow terms to be listed. Ignore any extra terms. Isw once a correct **simplified** answer is seen.

Note that M0M1A1A0 is a possible mark trait in either method

Note regarding a possible misread in parts (b) and (c)

Some candidates are misreading $\int \frac{6x}{(3+x)^2}\,\mathrm{d}x$ in parts (b) and (c) as $\int \frac{6}{(3+x)^2}\,\mathrm{d}x$

If parts (b) and (c) are consistently attempted with $\int \frac{6}{(3+x)^2}\,\mathrm{d}x$ then we will allow the M marks in (b) **only**. **M1** for $x^n \to x^{n+1}$ applied to their expansion in part (a) or 6× (their expansion in part (a)) and **dM1** for substituting in 0.4 and 0.2 and subtracting either way round (may be implied).
No marks are available in part (c)

MARK PARTS (b) and (c) TOGETHER

(b)	$\int 6x\times"\left(\frac{1}{9}-\frac{2x}{27}+\frac{x^2}{27}\right)"\,\mathrm{d}x = \int\left(\frac{2x}{3}-\frac{4x^2}{9}+\frac{2x^3}{9}\right)\mathrm{d}x = ...$	**M1**	1.1b
	$\int\left(\frac{2x}{3}-\frac{4x^2}{9}+\frac{2x^3}{9}\right)\mathrm{d}x = \frac{x^2}{3}-\frac{4x^3}{27}+\frac{x^4}{18}$ oe	**A1**	1.1b
	$\left["\frac{x^2}{3}-\frac{4x^3}{27}+\frac{x^4}{18}"\right]_{0.2}^{0.4} = \left(\frac{(0.4)^2}{3}-\frac{4(0.4)^3}{27}+\frac{(0.4)^4}{18}\right)-\left(\frac{(0.2)^2}{3}-\frac{4(0.2)^3}{27}+\frac{(0.2)^4}{18}\right)$	**dM1**	3.1a
	$=\text{awrt } 0.03304$ or $\frac{223}{6750}$	**A1**	1.1b
		(4)	

(b)
M1: Attempts to multiply their expansion from part (a) by $6x$ or just x and attempts to integrate. Condone copying slips and slips in expanding. Look for $x^n \to x^{n+1}$ at least once having multiplied by $6x$ or x. Ignore e.g. spurious integral signs.
A1: Correct integration, simplified or unsimplified.

$$\frac{x^2}{3}-\frac{4x^3}{27}+\frac{x^4}{18} \text{ oe e.g. } \frac{1}{9}\left(3x^2-\frac{4x^3}{3}+\frac{x^4}{2}\right),\ 6\left(\frac{x^2}{18}-\frac{2x^3}{81}+\frac{x^4}{108}\right)$$

If they have extra terms they can be ignored.
Ignore e.g. spurious integral signs.
dM1: An overall problem-solving mark for
- using part (a) by integrating $6x\times$ their binomial expansion and
- substituting in 0.4 and 0.2 and subtracting either way round (may be implied)

For evidence of using the correct limits we do not expect examiners to check so allow this mark if they obtain a value with (minimal) evidence of the use of the limits 0.4 and 0.2.
This could be e.g. $\left[\mathrm{f}(x)\right]_{0.2}^{0.4} = ...$ provided the first M was scored. If the integration was correct, evidence can be taken from answer of awrt 0.0330 if limits are not seen elsewhere. **Depends on the first M mark.**
A1: awrt 0.03304 (NB allow the exact value which is $\frac{223}{6750} = 0.033037037...$).
Isw following a correct answer.
Note answers which use additional terms in the expansion to give a different approximation score A0
Also note that the actual value is 0.032865…

Some may use integration by parts in (b) and the following scheme should be applied.

<u>Integration by parts in (b):</u>

Either by taking $u = 6x$ and $\dfrac{\mathrm{d}v}{\mathrm{d}x} = "\left(\dfrac{1}{9} - \dfrac{2x}{27} + \dfrac{x^2}{27}\right)"$

$$\int 6x \times "\left(\frac{1}{9} - \frac{2x}{27} + \frac{x^2}{27}\right)"\mathrm{d}x = 6x \times \left(\frac{1}{9}x - \frac{x^2}{27} + \frac{x^3}{81}\right) - 6\int\left(\frac{1}{9}x - \frac{x^2}{27} + \frac{x^3}{81}\right)\mathrm{d}x$$

$$= 6x\left(\frac{1}{9}x - \frac{x^2}{27} + \frac{x^3}{81}\right) - \left(\frac{1}{3}x^2 - \frac{2x^3}{27} + \frac{6x^4}{324}\right)$$

M1: A full attempt at integration by parts. This requires:

$$\int 6x \times "\left(\frac{1}{9} - \frac{2x}{27} + \frac{x^2}{27}\right)"\mathrm{d}x = kx \times \mathrm{f}(x) - k\int \mathrm{f}(x)\mathrm{d}x = kx \times \mathrm{f}(x) - k\mathrm{g}(x)$$

Where f(x) is an attempt to integrate their expansion from (a) with $x^n \to x^{n+1}$ at least once **and** g(x) is an attempt to integrate their f(x) with $x^n \to x^{n+1}$ at least once

A1: Fully correct integration. Then **dM1A1** as in the main scheme

Or by taking $u = "\left(\dfrac{1}{9} - \dfrac{2x}{27} + \dfrac{x^2}{27}\right)"$ and $\dfrac{\mathrm{d}v}{\mathrm{d}x} = 6x$

$$\int 6x \times "\left(\frac{1}{9} - \frac{2x}{27} + \frac{x^2}{27}\right)"\mathrm{d}x = 3x^2 \times \left(\frac{1}{9} - \frac{2x}{27} + \frac{x^2}{27}\right) - \int 3x^2 \times \left(-\frac{2}{27} + \frac{2x}{27}\right)\mathrm{d}x$$

$$= 3x^2 \times \left(\frac{1}{9} - \frac{2x}{27} + \frac{x^2}{27}\right) - \int\left(\frac{6x^3}{27} - \frac{6x^2}{27}\right)\mathrm{d}x = 3x^2 \times \left(\frac{1}{9} - \frac{2x}{27} + \frac{x^2}{27}\right) - \left(\frac{6x^4}{108} - \frac{6x^3}{81}\right)$$

M1: A full attempt at integration by parts. This requires:

$$\int 6x \times "\left(\frac{1}{9} - \frac{2x}{27} + \frac{x^2}{27}\right)"\mathrm{d}x = kx^2 \times \mathrm{f}(x) - k\int x^2\mathrm{g}(x)\mathrm{d}x = kx^2 \times \mathrm{f}(x) - k\mathrm{h}(x)$$

Where f(x) is their expansion from (a) and g(x) is an attempt to differentiate their f(x) with $x^n \to x^{n-1}$ at least once **and** h(x) is an attempt to integrate their $x^2\mathrm{g}(x)$ with $x^n \to x^{n+1}$ at least once

A1: Fully correct integration. Then **dM1A1** as in the main scheme

(c)	Overall problem-solving mark (see notes)	M1	3.1a
	$u = 3 + x \Rightarrow \int_{3.2}^{3.4} \mathrm{f}(u)\,\mathrm{d}u \Rightarrow \int_{3.2}^{3.4} \frac{6(u-3)}{u^2}\,\mathrm{d}u = \int_{3.2}^{3.4} \frac{6}{u} - \frac{18}{u^2}\,\mathrm{d}u \Rightarrow \ldots\ln u + \ldots u^{-1}$	M1	1.1b
	$\int_{3.2}^{3.4} \frac{6(u-3)}{u^2}\,\mathrm{d}u = \int_{3.2}^{3.4} \frac{6}{u} - \frac{18}{u^2}\,\mathrm{d}u \Rightarrow 6\ln u + 18u^{-1}$	A1	1.1b
	$\left[6\ln u + 18u^{-1}\right]_{3.2}^{3.4} = \left(6\ln 3.4 + \frac{18}{3.4}\right) - \left(6\ln 3.2 + \frac{18}{3.2}\right) = \ldots$	ddM1	1.1b
	$6\ln\left(\frac{17}{16}\right) - \frac{45}{136}$ oe	A1	2.1
		(5)	

(c) Alt 1	Overall problem-solving mark (see notes)	M1	3.1a
	$\int 6x(3+x)^{-2}\,\mathrm{d}x = \frac{\ldots x}{3+x} \pm \ldots\int (3+x)^{-1}\mathrm{d}x = \frac{\ldots x}{3+x} \pm \ldots\ln(3+x)$ oe	M1	1.1b
	$= 6\ln(3+x) - \frac{6x}{3+x}$ oe	A1	1.1b
	$\left(6\ln(3+0.4) - \frac{6(0.4)}{3+0.4}\right) - \left(6\ln(3+0.2) - \frac{6(0.2)}{3+0.2}\right) = \ldots$	ddM1	1.1b
	$6\ln\left(\frac{17}{16}\right) - \frac{45}{136}$ oe	A1	2.1

(c) Alt 2	Overall problem-solving mark (see notes)	M1	3.1a
	$\int 6x(3+x)^{-2}\,\mathrm{d}x = \int\left(\frac{\ldots}{(3+x)} + \frac{\ldots}{(3+x)^2}\right)\mathrm{d}x = \ldots\ln(3+x) + \frac{\ldots}{3+x}$ oe	M1	1.1b
	$= 6\ln(3+x) + \frac{18}{3+x}$ oe	A1	1.1b
	$\left(6\ln(3+0.4) + \frac{18}{3+0.4}\right) - \left(6\ln(3+0.2) + \frac{18}{3+0.2}\right) = \ldots$	ddM1	1.1b
	$6\ln\left(\frac{17}{16}\right) - \frac{45}{136}$ oe	A1	2.1

(13 marks)

Notes

(c) There are various methods which can be used

M1: An overall problem-solving mark for **<u>all of</u>**

- using an appropriate integration technique e.g. substitution, by parts or partial fractions – note that this may not be correct but mark positively if they have tried one of these approaches
- integrates one of their terms to a natural logarithm, e.g., $\frac{a}{3+x} \to b\ln(3+x)$ or $\frac{a}{u} \to b\ln u$
- substitutes in correct limits and subtracts either way round

M1: Integrates to achieve an expression of the required form for their chosen method

- substitution: $u = x + 3 \to \pm\frac{a}{u} \pm b\ln u$ or e.g. $u = (x+3)^2 \to \pm\frac{a}{\sqrt{u}} \pm b\ln u$
- parts: $\pm a\ln(3+x) \pm \frac{bx}{3+x}$ condone missing brackets e.g. $\ldots\ln x + 3$ for $\ldots\ln(3+x)$
- partial fractions: $\pm a\ln(3+x) \pm \frac{b}{3+x}$ condone missing brackets e.g. $\ldots\ln 3 + x$ for $\ldots\ln(3+x)$

A1: Correct integration for their method e.g.

- substitution: $u = x+3 \to 6\ln u + 18u^{-1}$ or e.g. $u = (x+3)^2 \to 3\ln u + \dfrac{18}{\sqrt{u}}$
- parts: $6\ln(3+x) - \dfrac{6x}{3+x}$
- partial fractions: $6\ln(3+x) + \dfrac{18}{3+x}$ oe e.g. $3\ln(9+6x+x^2) + \dfrac{18}{3+x}$

Note that the above terms may appear "separated" but must be correct with the correct signs.
(ignore any reference to a constant of integration)
Do not condone missing brackets e.g. $6\ln x + 3$ for $6\ln(3+x)$ unless they are implied by later work.

ddM1: Substitutes in the correct limits for their integral and subtracts either way round to find a value
Depends on both previous method marks.
For evidence of using the correct limits we do not expect examiners to check so allow this mark if they obtain a value with (minimal) evidence of the use of the appropriate limits.
This could be e.g. $\left[\mathrm{f}(x)\right]_{0.2}^{0.4} = ...$ provided both previous M marks were scored.
Note that for substitution they may revert back to $3 + x$ and so should be using 0.4 and 0.2

A1: A full and rigorous argument leading to $6\ln\left(\dfrac{17}{16}\right) - \dfrac{45}{136}$ or exact equivalent e.g. $3\ln\left(\dfrac{289}{256}\right) - \dfrac{45}{136}$ or e.g. $-6\ln\left(\dfrac{16}{17}\right) - \dfrac{45}{136}$

The brackets are not required around the $\dfrac{17}{16}$ and allow exact equivalents e.g. allow 1.0625 or $1\dfrac{1}{16}$ but not e.g. $\dfrac{3.4}{3.2}$. The $\dfrac{45}{136}$ must be exact or an exact equivalent. Also allow e.g. $6\ln\left|\dfrac{17}{16}\right| - \dfrac{45}{136}$
Ignore spurious integral signs that may appear as part of their solution.